An Introduction to Model Checking

Model checking has proven to be a successful technology to verify requirements and design for a variety of systems, particularly in hardware systems and real-time embedded and safety-critical systems. Here’s how it works.

Before you even start writing code on a project, you face the chronic problem of software development: flawed design requirements. Flawed requirements breed bugs that you have to get rid of later, often at much cost to the project and the bottom line, so it makes sense to find the flaws upfront.

In the last decade or so, the computer science research community has made tremendous progress in developing tools and techniques for verifying requirements and design. The most successful approach that’s emerged is called model checking. When combined with strict use of a formal modeling language, you can automate your verification process fairly well. In this article, I’ll introduce you to model checking and show you how it works.

Colossal task
It’s common to write design requirements heavily in English, interlaced with suitable tables, diagrams, screen shots, and UML descriptions, such as use cases and diagrams (for instance, sequence, class, and state transition diagrams).
When we check design requirements, we’re basically seeking the answers to a series of questions. Here are the general questions you’ll ask when checking your requirements:

- Do they accurately reflect the users’ requirements? Does everything you’ve stated match what the users want and have you included everything the users have requested?
- Are the requirements clearly written and unambiguous? Understandable?
- Are they flexible and realizable for the engineers? For instance, are the requirements suitably modular and well structured to aid in design and development?
- Can the requirements be used to easily define acceptance test cases to check the conformance of the implementation against the requirements?
- Are the requirements written in an abstract and high-level manner, away from design, implementation, technology platforms and so on, so as to give enough freedom to the designer and developers to implement them efficiently?

Finding the answers to these questions is a tall order and there’s no easy way to do it, but if your requirements can jump successfully through these hoops, you’ve got a good foundation for a sound system.

Despite some help from modeling tools such as UML, the problem of ensuring the quality of requirements remains. The process is heavily manual and time consuming, involving reviews and sometimes partial prototyping. Using multiple notations (such as those in UML) introduces additional problems:

- which notation to use for what requirements
- how to ensure that the descriptions in different notations are consistent with each other

The cost of errors in requirements are often high, requiring at least rework and maintenance. If you implement the incorrect requirements as they are, it may lead to incorrect system behavior in the field and high costs, such as loss of life and property, particularly in real-time, embedded safety-critical systems. Similar problems exist in ensuring the quality of system design.

One way to improve the quality of your requirements and design is to use automated tools to check the quality of various aspects of the requirements and design. But what tools? Building tools to check requirements or design written in English is clearly extremely difficult. It’s necessary to enforce a clear, rigorous, and unambiguous formal language for stating the requirements. If the language for writing requirements and design has well-defined semantics, it may be feasible to develop tools to analyze the statements written in that language. This basic idea using a rigorous language for writing requirements or design is now acknowledged as a foundation for system verification.

**Model checking**

Model checking is the most successful approach that’s emerged for verifying requirements. The essential idea behind model checking is shown in Figure 1. A *model-checking tool* accepts system requirements or design (called *models*) and a property (called *specification*) that the final system is expected to satisfy. The tool then outputs *yes* if the given model satisfies given specifications and generates a counterexample otherwise. The *counterexample*
For control-oriented systems, finite state machines (FSM) are widely accepted as a good, clean, and abstract notation for defining requirements and design. But a “pure” FSM is not adequate for modeling complex real-life industrial systems.

A simple system model

Let’s look at how you can use model checking to verify properties of a simple embedded system. We’ll use the symbolic model verifier (SMV) model-checking tool from Carnegie-Mellon University for this purpose. Of course, you can also write this model in most other model-checking tools. See the sidebar near the end of this article for a list of model-checking tools and where to find them.

Consider a simple pumping control system that transfers water from a source tank A into another sink tank B using a pump P, as shown in Figure 2. Each tank has two level-meters: one to detect whether its level is empty and the other to detect whether its level is full. The tank level is ok if it’s neither empty nor full; in other words, if it’s above the empty mark but below the full mark.

Initially, both tanks are empty. The pump is to be switched on as soon as the water level in tank A reaches ok (from empty), provided that tank B is not full. The pump remains on as long as tank A is not empty and as long as tank B is not full. The pump is to be switched off as soon as either tank A becomes empty or tank B becomes full. The system should not attempt to switch the pump off (on) if it’s already off (on). While this example may appear trivial, it easily extends to a controller for a complex network of pumps and pipes to control multiple source and sink tanks, such as those in water treatment facilities or chemical production plants.

The model of this system in SMV is as follows and is shown in Listing 1. The first $\text{VAR}$ section declares that the system has three state variables. Variables $\text{level}_a$ and $\text{level}_b$ record the current level of the upper and lower tank respectively; at each “instant” these variables take a value, which can be either empty, ok, or full. Variable $\text{pump}$ records whether the pump is on or off.

A system state is defined by a tuple of values for each of these three variables. For example, $(\text{level}_a=\text{empty}$,
level_b=ok, pump=off) and (level_a=empty, level_b=full, pump=on) are possible system states. The INIT section, near the end, defines initial values for the variables (here, initially the pump is assumed to be off but the other two variables can have any value).

The ASSIGN section defines how the system changes from one state to another. Each next statement defines how the value of a particular variable changes. All these assignment statements are assumed to work in parallel; the next state is defined as the net result of executing all the assignment statements in this section. The lower tank can go from empty to the empty or ok state; from ok to either empty or full or remain ok if the pump is on; from ok to either ok or full if the pump is off; from full can’t change state if the pump is off; from full to ok or full if the pump is on. Similar changes are defined for the upper tank.

Internally, most model-checking tools flatten (or unfold) an input model into a transition system called Kripke structure. Unfolding involves removing hierarchies in the EFSM, removing parallel compositions, and removing guards and actions on

**LISTING 1** An SMV model description and requirements list

```plaintext
MODULE main
VAR
  var: empty, ok, full; -- lower tank
  level_b: empty, ok, full; -- upper tank
  pump: on, off;
ASSIGN
  next(level_a) := case
    level_a = empty : {empty, ok};
    level_a = ok & pump = off : {ok, full};
    level_a = ok & pump = on : {ok, empty, full};
    level_a = full & pump = off : full;
    level_a = full & pump = on : {ok, full};
1 : {ok, empty, full};
esac;
next(level_b) := case
    level_b = empty & pump = off : empty;
    level_b = empty & pump = on : {empty, ok};
    level_b = ok & pump = off : {ok, empty};
    level_b = ok & pump = on : {ok, empty, full};
    level_b = full & pump = off : {ok, full};
    level_b = full & pump = on : {ok, full};
1 : {ok, empty, full};
esac;
next(pump) := case
    pump = off & (level_a = ok | level_a = full) &
    (level_b = empty | level_b = ok) : on;
    pump = on & (level_a = empty | level_b = full) : off;
1 : pump; -- keep pump status as it is
esac;
INIT
  (pump = off)
SPEC
  -- pump if always off if ground tank is empty or up tank is full
  -- AG AF (pump = off -> (level_a = empty | level_b = full))
  -- it is always possible to reach a state when the up tank is ok or full
  AG (EF (level_b = ok | level_b = full))
```
Each state in the Kripke structure is essentially a tuple containing one value for each state variable. A transition in Kripke structure denotes change in the value of one or more state variables. A given property is actually checked against the Kripke structure obtained by unfolding the given model. However, for the purposes of understanding what a property statement means, a Kripke structure is further unfolded into an infinite tree where each path in the tree indicates a possible execution or behavior of the system.

**TABLE 1** Some temporal connectives in CTL

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>EX φ</td>
<td>true in current state if formula φ is true in at least one of the next states</td>
</tr>
<tr>
<td>EF φ</td>
<td>true in current state if there exists some state in some path beginning in current state which satisfies the formula φ</td>
</tr>
<tr>
<td>EG φ</td>
<td>true in current state if there exists some state in every path beginning in current state which satisfies the formula φ</td>
</tr>
<tr>
<td>AX φ</td>
<td>true in current state if formula φ is true in every one of the next states</td>
</tr>
<tr>
<td>AF φ</td>
<td>true in current state if there exists some state in every path beginning in current state which satisfies the formula φ</td>
</tr>
<tr>
<td>AG φ</td>
<td>true in current state if every state in every path beginning in current state satisfies the formula φ</td>
</tr>
</tbody>
</table>

**FIGURE 3** Initial part of the execution tree for the pump controller system

**FIGURE 4** Intuition for CTL formulae which are satisfied at state $s_0$
The aim of model checking is to examine whether or not the execution tree satisfies a user-given property specification.

Paths and specs
Initially, the system could be in any of the nine states where there are no restrictions on the water level in A or B but the pump is assumed to be off. Let’s denote a state by an ordered tuple $<A,B,P>$ where $A$ and $B$ denote the current water level in tank $A$ and $B$, and $P$ denotes the current pump status. To illustrate, let’s assume the initial state to be $<\text{empty,empty,off}>$. Then as per the system model, the next state from this state could be any of the $<\text{empty,empty,off}>$, $<\text{ok,empty,on}>$. From $<\text{ok,empty,on}>$ the next state could be either of $<\text{ok,empty,on}>$, $<\text{ok,ok,on}>$, $<\text{full,empty,on}>$, $<\text{full,ok,on}>$, $<\text{empty,empty,off}>$, or $<\text{empty,ok,off}>$. For each of these states, we could calculate the next possible states.

The states can be arranged in the form of an infinite execution (or computation) tree, where the root is labeled with our chosen initial state and the children of any state denote the next possible states as shown in Figure 3. A system execution is a path in this execution tree. In general, the system has infinitely many such execution paths. The aim of model checking is to examine whether or not the execution tree satisfies a user-given property specification.

The question now is how do we specify properties of paths (and states in the paths) of an execution tree? Computation tree logic (CTL)—technically a branching time temporal logic—is a simple and intuitive notation suitable for this purpose. CTL is an extension of the usual Boolean propositional logic (which includes the logical connectives such as and, or, not, implies) where additional temporal connectives are available.

Table 1 and Figure 4 illustrate the intuitive meaning of some of the basic temporal connectives in CTL. Basically, $E$ (for some path) and $A$ (for all paths) are path quantifiers for paths beginning from a state, $F$ (for some state) and $G$ (for all states) are state quantifiers for states in a path.

Given a property and a (possibly infinite) computation tree $T$ corresponding to the system model, a model-checking algorithm essentially examines $T$ to check if $T$ satisfies the property. For example, consider a property $AF \ g$ where $g$ is a propositional formula not involving anyCTL connectives. Figure 4b shows an example of a computation tree $T$. The property $AF \ g$ is true for this $T$ if it’s true at the root state $s_0$—in other words, if there’s some state in every path in $T$ starting at $s_0$ such that the formula $g$ is true in that state. If $g$ is true at $s_0$ then we’re done, since $s_0$ occurs in every path starting at $s_0$. But suppose $g$ is not true in $s_0$. Then since every path from $s_0$ goes either to the left child or to the right child of $s_0$, the property is true at $s_0$ if it’s (recursively checked to be) true at both children of $s_0$ in $T$.

Figure 4b shows that $g$ is true at the root of the left subtree (indicated by the filled circle). Hence all paths from $s_0$ to left child and further down in the left subtree satisfy the property. Now suppose $g$ is not true at the right child of $s_0$, hence the property is recursively checked for all its children. Figure 4b shows that $g$ is true at all children of the right child of $s_0$ (indicated by filled circles) and hence the property is true for the right subtree of $s_0$. Thus the property is true for all subtrees of $s_0$ and hence it’s also true at $s_0$.

Figure 4 summarizes the similar reasoning used to check properties stated in other forms such as $EG \ g$ and $AG \ g$. Of course, in practice, the model-checking algorithms are really far more complex than this; they use sophisticated tricks to prune the state space to avoid checking those parts.
where the property is guaranteed to be true. Some good model checkers have been used to verify properties of state spaces of size as large as 1,040 states.

In SMV, a property to be verified is given by the user in the SPEC section. The logical connectives not, or, and, implies (if-then) are represented by $\neg$, $\lor$, $\land$, and $\rightarrow$ respectively. The CTL temporal connectives are AF, AG, EF, EG, and so on. The property AF ($pump = \text{on}$) states that for every path beginning at the initial state, there’s a state in that path at which the pump is on. This property is clearly false in the initial state since there’s a path from the initial state in which the pump always remains off (for example, if tank A forever remains empty). If this property is specified in the SPEC section, SMV generates the following counterexample for the property. The loop indicates an infinite sequence of states (in other words, a path) beginning at the initial state such that tank B is full in every state of the path and hence pump is off.

```
-- specification AF pump = on is false
-- as demonstrated by the following execution sequence
-- loop starts here
state 1.1: level_a = full
state 1.2: level_b = full
```

The dual property AF ($pump = \text{off}$) states that for every path beginning at the initial state, there’s a state in that path at which the pump is off. This property is trivially true at the initial state, since in the initial state itself (which is included in all paths) $pump = \text{off}$ is true.

You can specify interesting and complex properties by combining temporal and logical connectives. The property $\text{AG} ((\neg pump = \text{off}) \rightarrow \text{AF} (pump = \text{on}))$ states that it’s always the case that if pump is off then it eventually becomes on. This property is clearly false in the initial state. The property $\text{AG} \text{AF} (pump = \text{off} \rightarrow (\text{level}_a = \text{empty} \lor \text{level}_b = \text{full}))$ states that pump is always off if ground tank is empty or the upper tank is full. The property $\text{AG} (\text{EF} (\text{level}_b = \text{ok} \lor \text{level}_b = \text{full}))$ states that it’s always possible to reach a state when the upper tank is ok or full.

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### Model checking tools

**PUBLIC DOMAIN**

The following are some (but not all) prominent public domain model-checking tools, many of which are freely available for noncommercial use and can be used commercially by paying a license fee (check the web sites for detailed conditions).

<table>
<thead>
<tr>
<th>MODELING CHECKING TOOL</th>
<th>MODELING</th>
<th>SPECIFICATION</th>
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<tbody>
<tr>
<td>SMV</td>
<td>Network of automata that communicate using shared variables</td>
<td>CTL</td>
</tr>
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<td>Control instruction set</td>
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<td>Coloured Petri Nets</td>
<td>CTL</td>
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<td><a href="http://www.daimi.au.dk/designCPN">www.daimi.au.dk/designCPN</a></td>
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</table>

**COMMERCIAL**

Some (again, certainly not all) commercial model checking tools are:

Every model-checking tool comes with its own modeling language that provides no way to automatically translate informal requirement descriptions into this language.

Model checking in practice

Model checking has proven to be a tremendously successful technology to verify requirements and design for a variety of systems, particularly in hardware systems and real-time embedded and safety-critical systems. For example, the SPIN model-checker was used to verify the multi-threaded plan execution module in NASA’s DEEP SPACE 1 mission and discovered five previously unknown concurrency errors.3

However, there are some major issues to deal with when using model checking in practice. For example:

- Every model-checking tool comes with its own modeling language that provides no way to automatically translate informal requirement descriptions into this language. This translation is necessarily manual and hence it’s difficult to check whether the model correctly represents your system. In fact, there may be parts of your requirements that may be difficult or even impossible to model in the given modeling notation.
- Similar problems exist for the tool-specific property specification notation, which is often a variant of CTL, CTL*, or propositional linear temporal logic (PLTL). Some properties to be verified may be difficult or even impossible to express in the notation.
- The number of states in your model may be extremely large. Although model-checking algorithms include ingenious ways to reduce this state space, the model checker may still take too long to verify a given property or “give up” during this task. In such cases the user has to put in more work, such as verifying parts of the model separately or reducing the state space by reducing domains of variables.

Nevertheless, model checking is likely to prove an invaluable way to verify system requirements or design. It often leads to early detection of the shortcomings in the requirements or design, thereby leading to large savings in later rework.

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--Girish Palshikar

References


Further reading